Comment on the Quantum Brachistochrone Problem

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In this brief comment we attempt to clarify the apparent discrepancy between the papers [1] and [2] on the quantum brachistochrone, namely whether it is possible to use a judicious mixture of Hermitian and non-Hermitian quantum mechanics to evade the standard lower limit on the time taken for evolution by a Hermitian Hamiltonian with given energy dispersion between two given states.

We do not dispute the theorem of Ref. [2], which is a generalization of Ref. [3] and states that the limit can not be evaded for unitary time development, but nonetheless maintain that the effect identified in Ref. [1] is a real one. How, then, is the theorem evaded? This can be explained in two ways. First one could use the framework of standard quantum mechanics to describe the entire process: preparation of the initial state, time development under the influence of the non-Hermitian Hamiltonian, and analysis of the final state. In that case, clearly the time development is not unitary in the usual sense, hence the possibility of an improved result. In this context the importance of the PT symmetry of the intermediate, non-Hermitian Hamiltonian is that its energy eigenvalues are real, so that the condition of the standard limit, namely that the energy dispersion be fixed, makes sense.

The second way of explaining the improvement on the standard lower limit is to use the alternative CPT framework of quantum mechanics for the time-development, whereby the intermediate Hamiltonian is Hermitian, and hence the time-development unitary, with respect to a new metric. The lower limit is now avoided because the initial and final states, which we chose to be orthogonal with respect to the standard metric, are no longer orthogonal with respect to this new metric. However, in retrospect it is to be admitted that we made too liberal use of the term unitary in our original paper, because this unitarity

was only with respect to the CPT operator defined for the intermediate Hamiltonian. In the set-up where this intermediate Hamiltonian represents a "black box" in an otherwise Hermitian world, true unitarity would have to refer to the C-operator for the entire set-up, something we did not calculate.

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[1] C. M. Bender, D. C. Brody, H. F. Jones and B. K. Meister, Phys. Rev. Lett. **98**, 040403 (2007).

[2] A. Mostafazadeh, Phys. Rev. Lett. **99**, 130502 (2007)

[3] D. C. Brody, J. Phys. A: Math. Gen. **36**, 5587 (2003).